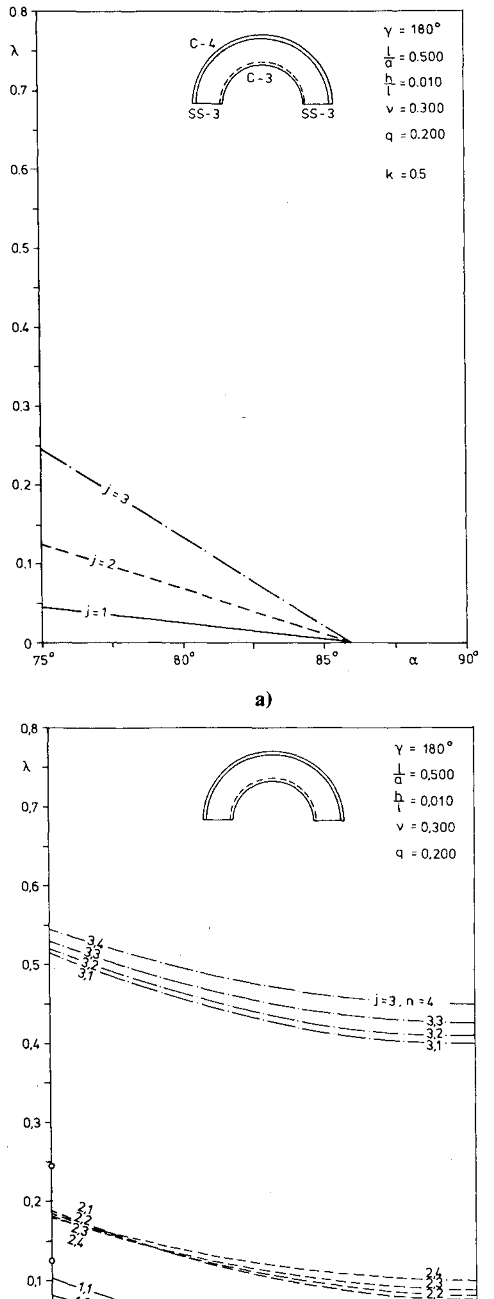

Fig. 4 Eigenvalues λ of a shallow conical shell as a function of vertex half-angle α.

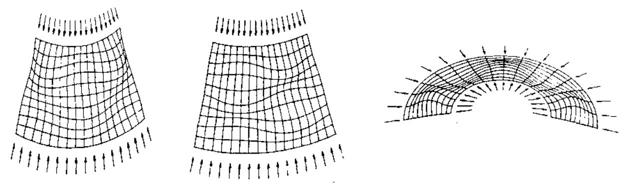

Fig. 5 Mode shapes of conical shell panels under compression.

panels with different boundary conditions. Comparison with more accurate numerical calculations shows that the approximation gives good results for the frequency- and load-range in which a conical shell neither vibrates in resonance nor buckles.

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Static Instability of an Elastically Restrained Cantilever Under a Partial Follower Force

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Introduction

THE stability of a cantilever under a force influenced by a parameter η (see Fig. 1) was examined by Dzhanelidze.[1] He found that, particularly within the range $0 \leq \eta \leq 0.5$, the static criterion of stability remains valid, i.e., the loss of stability occurs by divergence and not by flutter. Sundararajan[2] considered a uniform, clamped, elastically supported column subjected to a tangential load at the elastically supported end. He found that the critical instability mechanism changed from flutter to divergence, or vice versa, with a change in the stiffness of the elastic supports. Kounadis[3] considered simple structure elastically restrained under follower compressive forces. He pointed out that the type of instability in the structures is also dependent on the values of the constants of elastic restraint. In Ref 4, Kounadis presented a thorough discussion of the influence of the parameters of the elastic end constraints on the type of instability and estimated the smallest critical load of a column under tangential compressive force using a static stability criterion. The stiffness constant of a rotational or translational spring at one end of the column was assumed to be a parameter.

In this Note, the influence of the follower parameter η and an elastic end support on the divergence instability of the column will be investigated. The divergence instability of the column mentioned above subjected to the load changing from a constant direction ($\alpha = 0$) to a purely tangential one ($\alpha = 1$), is taken into consideration. The basic analytical method used by Kounadis[4] has been adopted here.

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Table 1 Extrema of the functions $c=c(k,\eta)$

η	0.2		0.3		0.4		0.5	0.6	0.7	0.8	0.9	1.0
k_0	4.2398	4.7290	3.8244	5.0404	3.4941	5.2239	5.3633	5.4757	5.5683	5.6451	5.7093	5.7630
c_0	12.0350	20.9162	5.8208	26.1791	2.3718	28.8391	30.6236	31.9227	32.9140	33.6873	34.3065	34.815
$\frac{d^2c}{dk^2}$	−63.919	85.980	−18.246	41.140	−9.2624	33.286	30.382	29.540	29.885	31.0138	32.7039	34.809

Table 2 Variability of functions $c=c(k)$ with parameter η

Parameter	$0\le c\le c^*$	$c^*<c<\infty$
$0\le\eta<\eta^*$	$\frac{dc}{dk}>0$	$\frac{d^2c}{dk^2}>0$
$\eta=\eta^*$	$\frac{dc}{dk}>0;\ \frac{d^2c}{dk^2}>0$	$k=k^*$
$\eta^*<\eta\le 0.5$	$\left.\frac{dc}{dk}\right\|_{k=k_0}=0;\ \frac{d^2c}{dk^2}<0$	$\left.\frac{dc}{dk}\right\|_{k=k_0}=0;\ \frac{d^2c}{dk^2}>0$
$0.5<\eta\le 1$	Nonexistent	

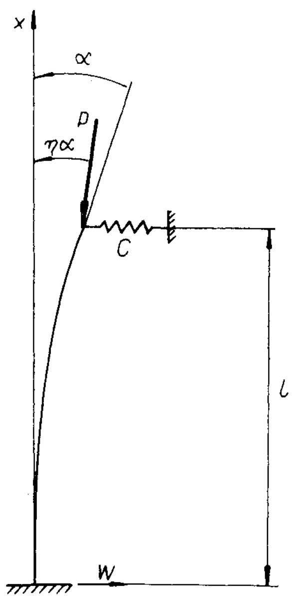

Fig. 1 Clamped elastically supported column under partial follower force.

Analysis

Consider a uniform, clamped, elastically supported column subjected to a follower force P (Fig. 1) in proportion to the parameter η. According to the Bernoulli-Euler theory, the differential equilibrium equation in nondimensional form may be written as

$$w,_{\xi\xi\xi\xi}+k^2w,_{\xi\xi}=0 \tag{1}$$

The following dimensionless quantities are introduced:

$$k^2=P\ell^2/EJ,\quad c=C\ell^3/EJ,\quad \xi=x/\ell,\quad w=W/\ell \tag{2}$$

where W is the transverse displacement, EJ the flexural rigidity, C the stiffness of the elastic support and, ℓ the length of the column. The boundary conditions are

$$w=w,_{\xi}=0 \qquad \text{at } \xi=0 \qquad (3a,3b)$$

$$w,_{\xi\xi}=w,_{\xi\xi\xi}+(1-\eta)k^2w,_{\xi}-c\cdot w=0 \quad \text{at } \xi=1 \qquad (3c,3d)$$

The solution of Eq. (1) satisfying the conditions of Eqs. (3a) and (3c) is

$$w=-\frac{M_0}{P}+\left(w_1+\frac{M_0}{P}\right)\xi+\frac{M_0}{P}\,\frac{\sin k(1-\xi)}{\sin k} \tag{4}$$

where

$$M_0=-\frac{1}{\ell^2}EJw,_{\xi\xi}(0) \quad\text{and}\quad w_1=w(1)$$

Results

Taking into account the two remaining boundary conditions [Eqs. (3b) and (3d)] to which the solution of Eq. (4) is subjected, the buckling equation is simultaneously the equation of the monoparametric family of curves and is obtained as follows:

$$(k\cos k-\sin k)c-[1-(1-\eta)(1-\cos k)]k^3=0 \tag{5}$$

where η is a parameter. It is worth mentioning that the curves from the region $k>0$ and $c>0$ have been considered. Also, the analytical considerations as well as the numerical calculations have been restricted to the particular curves of Eq. (5) that correspond to the first and second buckling loads.

The derivative of the function defined by Eq. (5) is presented in the following form:

$$\frac{dc}{dk}=\frac{Q(k,c)}{P(k)} \tag{6a}$$

where

$$Q(k,c)=\left[\left(\frac{3}{k}+ctg\frac{k}{2}\right)(k\cos k-\sin k)+k\sin k\right]c-k^3ctg\frac{k}{2} \tag{6b}$$

$$P(k)=k\cos k-\sin k \tag{6c}$$

Therefore, the differential equation (6a) of the family of curves of Eq. (5) has been obtained. Every point (k^*,c^*) for which the following relationships hold

$$P(k^*)=Q(k^*,c^*)=0 \qquad (7a,7b)$$

is then the critical point of differential equation (6a). In the case considered here, the point $S(k^*,c^*)$, where k^* is the smallest real positive root of Eq. (7a), is then the saddle point (cf. Ref. 5).

The remaining c^* coordinate is determined from Eq. (7b), while the value of η^* corresponding to point $S(k^*,c^*)$ evaluated from Eq. (5). The coordinates k^* and c^* as well as the parameter η^* satisfy the following equations:

$$k^*=tgk^* \tag{8a}$$

$$c^*=k^*(1-\eta^*) \tag{8b}$$

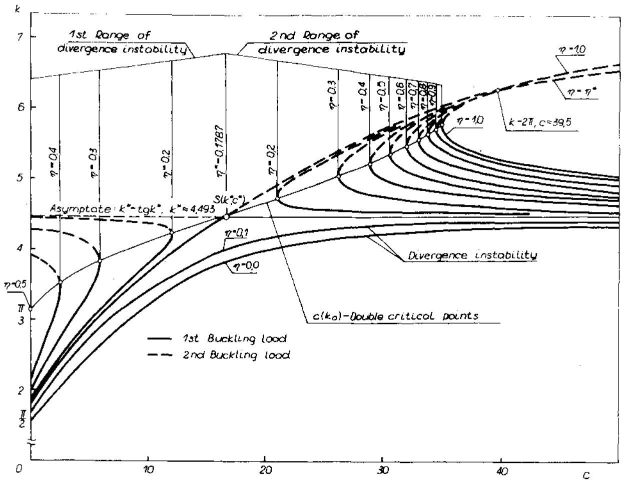

Fig. 2 Regions of divergence instability for clamped, elastically supported column under force that follows according to parameter η.

$$\eta^* = -\cos k^* / (1 - \cos k^*) \tag{8c}$$

From an analysis of Eq. (6a) and its general solution [Eq. (5)], one can easily find that the functions $c = c(k,\eta)$ have their extrema $c_0(\eta)$ at points $k = k_0$, with points (k_0, c_0) belonging to curve $Q(k,c) = 0$ [with $P(k) \neq 0$]. The curves belonging to the family of Eq. (5) and curve $c(k_0)$ (double critical points) are shown in Fig. 2. The characteristics of the Eq. (5) curves are listed in Tables 1 and 2.

Considering the results obtained by Kounadis,[4] the region of divergence instability can be determined. If $c_0(\eta)$ is a maximum of Eq. (5), divergence instability occurs in the region $0 \leq c \leq c_0(\eta)$; if c_0 is a minimum, divergence instability takes place in the region $c_0(\eta) \leq c < \infty$. In the problem considered above, this occurs for values of the parameter $\eta > \eta^*$. In the case $\eta < \eta^*$, divergence instability occurs independently of the rigidity of a column support characterized by c (cf. Fig. 1 and Table 2). The first buckling load for column at $\eta = \eta^*$ and $c > c^*$ is the equivalent of the critical force of a clamped, hinged column. For $\eta^* < \eta \leq 0.5$, divergence instability takes place within two ranges depending on the rigidity c of a column support. On the other hand, for $0.5 < \eta \leq 1$, divergence instability occurs within one range for $c > c_0(\eta)$ (Fig. 2 and Table 2).

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Efficient Repetitive Solution of Linear Equations with Varying Coefficients

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FINITE element analysis often deals with problems that involve the repetitive solution of a set of linear equations with both fixed and variable coefficients. Examples include the optimization of structures, electrical circuits, fluids models, etc. A method for the efficient solution of such sets of equations is developed and substantial reductions in effort are demonstrated. Savings can easily approach one or two orders of magnitude in the computational effort required. Equations are explicitly stated and operation count estimates are made.

This method is especially useful for equation sets containing a sufficient number of degrees of freedom to prohibit solutions on the user's digital computer because of time and cost constraints. The procedures suggested will make possible the solution to many problems that would otherwise require larger and more expensive computers. The mathematics are based upon an assumed parametric functional relationship in the coefficient matrix; however, the technique may be employed with other parametric relationships to derive similar results.

The linear equations are ordered as follows: 1) degrees of freedom with constant coefficients (k set) and 2) degrees of freedom with variable coefficients (v set). Variable terms in the coefficient matrix are removed prior to factoring and carried as auxiliary diagonal matrices. Factoring a matrix the size of the original coefficient matrix is required only once. Repetitive solutions require the factoring of a matrix equal in size to the number of degrees of freedom in the v set. The operation count savings for matrix factoring is on the order of the ratio of the above matrix sizes cubed.

The linear system to be solved is assumed to have the form

$$MX = B \tag{1}$$

The partitioned matrix M is defined by

$$M = \begin{bmatrix} A_{kk} & A_{kv} \\ A_{vk} & A_{vv} \end{bmatrix} \begin{bmatrix} I & 0 \\ 0 & E \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & D \end{bmatrix} \tag{2}$$

The A matrix of Eq. (2) has constant coefficients only. Parametric variations are achieved by manipulation of the auxiliary diagonal E and D matrices. Substitution of Eq. (2) into Eq. (1) yields

$$\left[\begin{bmatrix} A_{kk} & A_{kv} \\ A_{vk} & A_{vv} \end{bmatrix} \begin{bmatrix} I & 0 \\ 0 & E \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & D \end{bmatrix} \right] \begin{bmatrix} X_k \\ X_v \end{bmatrix} = \begin{bmatrix} B_k \\ B_v \end{bmatrix} \tag{3}$$

The A matrix of Eq. (3) may now be expressed as the product of its lower and upper triangle of factors, $A = LU$. Expressing the factors in partition form and substituting into Eq. (3) yields

$$\begin{bmatrix} L_{kk} & 0 \\ L_{vk} & L_{vv} \end{bmatrix} \begin{bmatrix} U_{kk} & U_{kv}E \\ 0 & U_{vv}E \end{bmatrix} \begin{bmatrix} X_k \\ X_v \end{bmatrix} + \begin{bmatrix} 0 \\ DX_v \end{bmatrix} = \begin{bmatrix} B_k \\ B_v \end{bmatrix} \tag{4}$$

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